STOCHASTIC RUNGE–KUTTA METHOD FOR STOCHASTIC DELAY DIFFERENTIAL EQUATIONS

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To my beloved husband

Abdul Hakim Mohd Amin

Mother and Father

Rusnani Yusof and *Rosli Yusoff*

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ABSTRACT

Random effect and time delay are inherent properties of many real phenomena around us, hence it is required to model the system via stochastic delay differential equations (SDDEs). However, the complexity arises due to the presence of both randomness and time delay. The analytical solution of SDDEs is hard to be found. In such a case, a numerical method provides a way to solve the problem. Nevertheless, due to the lacking of numerical methods available for solving SDDEs, a wide range of researchers among the mathematicians and scientists have not incorporated the important features of the real phenomena, which include randomness and time delay in modeling the system. Hence, this research aims to generalize the convergence proof of numerical methods for SDDEs when the drift and diffusion functions are Taylor expansion and to develop a stochastic Runge–Kutta for solving SDDEs. Motivated by the relative paucity of numerical methods accessible in simulating the strong solution of SDDEs, the numerical schemes developed in this research is hoped to bridge the gap between the evolution of numerical methods in ordinary differential equations (ODEs), delay differential equations (DDEs), stochastic differential equations (SDEs) and SDDEs. The extension of numerical methods of SDDEs is far from complete. Rate of convergence of recent numerical methods available in approximating the solution of SDDEs only reached the order of 1.0. One of the important factors of the rapid progression of the development of numerical methods for ODEs, DDEs and SDEs is the convergence proof of the approximation methods when the drift and diffusion coefficients are Taylor expansion that had been generalized. The convergence proof of numerical schemes for SDDEs has yet to be generalized. Hence, this research is carried out to solve this problem. Furthermore, the derivative-free method has not yet been established. Hence, development of a derivative–free method with 1.5 order of convergence, namely stochastic Runge–Kutta, to approximate the solution of SDDEs with a constant time lag, $r > 0$, is also included in this thesis.

ABSTRAK

Kesan rawak dan masa lengahan adalah ciri-ciri yang dipunyai oleh kebanyakan fenomena di sekeliling kita. Maka fenomena ini perlu dimodelkan menggunakan persamaan pembezaan stokastik lengahan (SDDEs). Walaubagaimanapun, kerawakan dan masa lengahan menyebabkan persamaan pembezaan bertambah rumit. Penyelesaian analitik SDDEs sukar untuk dicari. Bagi kes tersebut, kaedah berangka menyediakan cara untuk menyelesaikan masalah yang terlibat. Namun, disebabkan oleh kekurangan kaedah-kaedah berangka yang sedia ada untuk menyelesaikan SDDEs, ramai penyelidik dari kalangan ahli matematik dan saintis tidak memasukkan ciri-ciri penting fenomena nyata iaitu kesan rawak dan masa lengahan dalam memodelkan sistem tersebut. Maka, kajian ini bertujuan untuk mengitlakkan pembuktian penumpuan kaedah-kaedah berangka SDDEs apabila fungsi hanyutan dan resapan merupakan pengembangan Taylor dan membangunkan kaedah berangka stokastik Runge–Kutta untuk menyelesaikan SDDEs. Dimotivasikan oleh kekurangan relatif kaedah-kaedah berangka yang boleh diakses dalam simulasi penyelesaian kukuh SDDEs, skema-skema berangka yang dibangunkan diharap dapat merapatkan jurang di antara perkembangan kaedah-kaedah berangka persamaan pembezaan biasa (ODEs), persamaan pembezaan lengahan (DDEs), persamaan pembezaan stokastik (SDEs) dan SDDEs. Perkembangan kaedah-kaedah berangka SDDEs adalah jauh ketinggalan. Kadar penumpuan kaedah-kaedah berangka yang boleh didapati kini bagi menghampirkan penyelesaian SDDEs hanya mencapai peringkat 1.0. Salah satu daripada faktor-faktor penting perkembangan pesat pembangunan kaedah-kaedah berangka untuk ODEs, DDEs dan SDEs ialah pembuktian penumpuan kaedah-kaedah penghampiran apabila pekali-pekali hanyutan dan resapan merupakan kembangan Taylor yang telah diitlakkan. Pembuktian penumpuan kaedah-kaedah berangka SDDEs masih belum diitlakkan. Maka, kajian ini dijalankan untuk menyelesaikan masalah tersebut. Tambahan pula, kaedah bebas terbitan belum pernah dibangunkan. Maka, pembangunan kaedah bebas terbitan dengan kadar penumpuan 1.5, iaitu stokastik Runge–Kutta, untuk menghampirkan penyelesaian SDDEs dengan masa lengahan malar, $r > 0$, juga telah dimuatkan di dalam tesis ini.